# The Special Theory of Relativity

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"*Newton, forgive me; you found the only way which in your age, was just about possible for a man of highest thought and creative power. The concepts, which you created, are even today still guiding our thinking in physics, although we now know that they will have to be replaced by others farther removed from the sphere of immediate experience, if we aim at a profounder understanding of relationships.*"[1]

This is how Albert Einstein had expressed his feelings about the conceptual departure from Newton when he had embarked on his theory of relativity (TOR). And he had forced profound changes in the conceptions of space, time, length, duration, motion, mass, gravitation, and so on, in a word in all the fundamental aspects of the Newtonian mechanics.

What are these changes? Why were they necessary at all? How do they affect our conception of the world around us? These are the questions to be addressed to in this article, which will be concerned with the special theory of relativity only.

### From Aristotle to Galileo

In order to have a proper grasp of the matter, that is, the scientific background of the emergence of the TOR, we have to start from exactly where it had actually started. Just as Galileo is considered the father of modern physics, so also Aristotle is considered the father of the science of physics as such, including the title of this subject. Today almost all his ideas and theories are held as wrong; still the fact remains that he had started the systematic observation of the material world and catalogued them under certain physical rules.

One such rule involved the relation between force and motion. Like his contemporaries he had rightly observed from day to day experiences that a body tends to move as long as it is guided by an external force acting on it. For example, shooting of arrows, throwing of spears, pulling of carts, lifting of buckets of water, etc. Withdraw the action, and the body will gradually slow down and then stop at a distance. The greater the force, the farther the body moves. If there were no external action, the body would be at absolute rest. So Aristotle with all his genius generalized these observations into an intuitive thesis that a body moves only when it is exerted by an external force, or, *force creates motion*. Motion is the result of the acts like pushing, pulling, lifting, throwing, and so on. For nearly two thousands of years mankind accepted this thesis as the most obvious truth about motion. It was thought of as the true or absolute motion of the body in question. The tremendous authority of Aristotle lent much weight to this intuitive belief.

In the sixteenth century, these ideas about absolute motion and rest were first tacitly challenged in the Copernicus' theory of the earth's rotation, which rejected the daily rotation of the sun as its true motion and sought to explain it as an *apparent* motion as viewed from the earth. It showed the earth to be in a state of apparent rest with respect to the things on it. Then came Galileo, who laid bare

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the implicit idea of Copernicus and made the real and final conceptual break with the Aristotelian idea of absolute motion and rest. In the words of Einstein: "The method of reasoning dictated by intuition was wrong and led to the false ideas of motion which were held for centuries.... The discovery and use of scientific reasoning by Galileo was one of the most important achievements in the history of human thought, and marks the real beginning of physics."[2]

What were the essential points of Galileo's reasoning?

The first thing he enunciated was that the motion encountered with in daily life did not represent true or absolute motion, but apparent or relative motion of the bodies concerned. He in fact wanted to argue that this relative motion was the only, and therefore, the true motion of a body one could actually find, or, meaningfully deal with. Absolute motion, if any, was beyond the reach of man's handling or comprehension. And by relative motion of a body he meant its movement as seen and measured in comparison to another body. In modern parlance this is called the study of motion with respect to a frame of reference, the simplest form of which is a Cartesian a co-ordinate system.

To clinch the point he urged the people to give up the false lead of the empirical observations and probe deeper into the problem of motion. For this purpose he used to give a simple ideal example: Sitting inside a ship floating on a calm and quiet sea you cannot tell by any experiment (like throwing a ball in any direction, dropping a stone from above downwards, etc.) whether the ship is moving at a constant speed in a direction, or is at rest. You could tell it only if you had a scope to see the coastline, or the night sky (a frame of reference), and compare the position of the ship with that. And he argued that for a similar kind of situation we could not feel or understand the daily spin or yearly rotation of our beloved earth. If we could set our viewing position on the moon, or the sun, we could definitely see the earth moving.

From this he came to the second point. The relative state of uniform motion of a body, that is, its movement at a constant speed and the state of its relative rest can be distinguished only in terms of the frames of reference in which they are studied, and not in terms of any force. The state of rest and the state of uniform motion are equivalent concepts and none of them is the result of any external action. A body left to itself without any external force acting on it will be in the same state – at rest or in uniform motion, depending upon the co-ordinate systems chosen.

Thus, whereas for Aristotle a constant velocity requires a constant force, for Galileo a constant velocity implies the absence of any force as such. Totally opposing claims!

Then how do we detect the action of a force on a body? Galileo showed, the action of a force on a body is to *change* the rate of motion, that is, velocity and not to *create* it. By this change of velocity he meant acceleration. So he modified the thesis of Aristotle as follows: *Force leads to change of motion, that is, acceleration.*

For example, he asked, why do moving bodies slow down and finally stop when no external action is made to drive it? Is it because no force is acting on it? On the contrary, he showed, some forces are working on it in the opposite direction – for example, the frictional forces of the surface on which it is moving, the resistance of the air, etc., which make it slow down and ultimately stop. Smoothen the surface, reduce the friction as much as you can, remove air, and you will see the body move farther and farther. If you could have an ideal surface free from all kinds of friction and in vacuum, you would see the body move forever.

In only one special case is still Aristotle defensible. When a body moves in a viscous medium it requires a constantly acting external force. In fact, even a uniform motion of this kind does not take place in a force-free condition. And this is the kind of motion we usually meet in our daily life. The great ancient philosopher was not aware that he had turned the property of this special case into a rule for motion in general.

Einstein summarized the discovery thus: "[The] action of an external force changes the velocity. Thus not the velocity itself but its change is a consequence of pushing or pulling. Such a force either increases or decreases the velocity according to whether it acts in the direction of motion or in the opposite direction…. By following [this] right clue we achieve a deeper understanding of motion. The connection between force and the change of velocity – and not, as we think according to our intuition, the connection between force and the velocity itself – is the basis of classical mechanics as formulated by Newton."[3]

## Galilean Relativity and Newtonian Mechanics

This is where Newton took over. Galileo had already introduced reasoning, experiment, thought experiment, mathematics, etc. into physics. He made many important contributions in physics and astronomy. He was also well accomplished in arts, literature, music, paintings, etc. We are not going to concern us over them here but shall pass on straight to the relativity principle deduced by Newton from the above concepts of motion as developed by Galileo.

Let us note in passing that the concept of relativity here connotes relativity of motion, or study of dynamics of bodies in relation to certain co-ordinate systems. Newton first applied his newly invented tool of fluxions (now called differentials) into the Galilean ideas. The fact that force results in a change of velocity was stated by him as follows:

$$F = m.dv/dt = m.d^2s/dt^2, \qquad (1)$$

where $m$ is the mass of the body, $v$ is the velocity at time $t$, and $ds$ is the displacement in time $dt$.

Note that according to Aristotle, this equation should have been written as $F = m.ds/dt$, which involves an error in the order of the derivative.

The differential part of the right hand side of (1) for a given body represents acceleration, which is, say, = $a$. Then he wrote this in a derivative form $dv/dt = a$, which on integration yields,

$$v = u + at, \qquad (2)$$

where $u$ is the constant of integration.

From this result it is obvious that if $a = 0$, $v = u$.

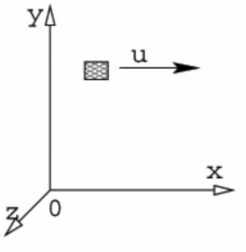

Fig.1a

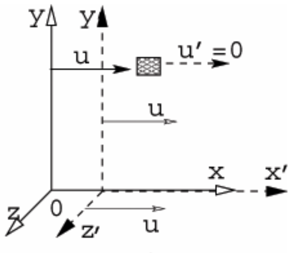

Fig.1b

Newton showed that in one frame of reference the body may appear to move with a constant velocity of $u$ (see fig. 1a); whereas, in another frame of reference, which is itself moving with a uniform velocity of $u$ compared to the first frame, the same body will appear to be at rest, that is, its velocity in this frame of reference, $v = 0$ (see fig. 1b). Moreover, in both cases $F = 0$. This shows that the state of rest and the state of uniform motion are equivalent, both being in a force-free state, and are related only to the frames of reference chosen.

It is also clear from (2) that this law of motion is valid in both these frames of reference. And, at the same time, both the frames are equivalent for this law. Newton then found that all the laws of motion are valid in these two types of frames of reference, and all frames of reference, which move with constant velocities with respect to one another, are equivalent for these laws. He termed the tendency of a body to continue in the same state (of relative rest or relative motion) in a force-free condition as *inertia*. And he called these frames of reference, which show a body to be in either of these states, as *inertial frames of reference*.

This new knowledge about the motion of bodies led to the Galilean Principle of Relativity, which can be stated as: The laws of mechanics are valid in all inertial frames of reference. According to this principle there is no preferred or absolute frame of reference, which would reveal the absolute motion of a body. In order to apply the laws of motion from one frame to another, it is only necessary to make use of some transformations laws (known since 1909 after Phillip Frank as Galilean transformation). If the position co-ordinates of a body in one frame of reference are given as $x$, $y$, $z$, and, those in another frame moving with a uniform velocity $v$ along the horizontal ($X$-$Z$) plane after time $t$, as $x'$, $y'$, $z'$ (see fig. 2), then, the corresponding co-ordinates will be related as:

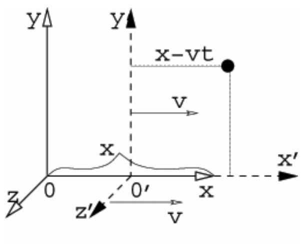

Fig.2

$$\begin{aligned} \underline{x'} &= x - vt, \\ y' &= y \\ z' &= z \\ t &= t' \end{aligned} \tag{3}$$

The fourth equation in this series of transformations was not required or used in the original formulation. We have written them in order to show their relations to the next stage of development in the relativity principle. It was, however, taken for granted in the Newtonian mechanics that measurement of time in any two co-ordinate systems uniformly moving with respect to each other did not vary (that is, the time axis of the second frame could in no way differ from that of the first). It was also tacitly assumed that the time axis was in no way related to the space axes but existed independently of them.

Also note that this is a simple representation of the relativity principle. It can be easily generalized for the case when the second frame moves at an angle $\theta$ with the $X$-$Z$ plane (fig. 3), using suitable trigonometric relations, for example, $x' = x - vt\cos\theta$, $y' = y - vt\sin\theta$, $z = z'$, etc., and still a step further, where $z'$ will also differ from $z$. But no further, for, it was not considered possible to visualize a movement of the frame of reference such that $t'$ would also differ from $t$.

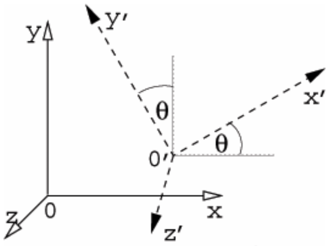

Fig.3

The Newtonian mechanics, which was created on the basis of this Galilean relativity principle, considered the space and time dimensions it dealt with and measured to be relative. Since velocity, which is a function of spatial and temporal measurements, is relative, the space and time measured also cannot but be relative. However, in the Newtonian system the length of a body and the duration of an event were observed as invariant in all frames of reference – and therefore considered to be absolute. So Newton did not rule out the existence of a body in a far away corner of the universe, which would be at absolute rest. He wrote: "It is possible that, in the remote regions of the fixed stars, or perhaps far beyond them, there may be some body absolutely at rest; but it is impossible to know, from the position of bodies to one another in our region, whether any of these do keep the same position relative to that remote body."[4] Similarly, Newton, while believing in a universal flow of time independent of any frame of reference, viewed the time measurement as relative in the sense of being *arbitrary*, its units being chosen by human convention and convenience, and thought that the measurement of time would give absolute values in the supposedly far-off object.

Newton held the relative measurements of space and time as subjective, as merely apparent, as against the objective concepts of space and time, which were according to him absolute. He wrote: "Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces; which our senses determine by its position to bodies, and which is commonly taken for immovable space." As regards time he argued: "Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year."[5]

Newtonian mechanics was tremendously successful. It combined the Galilean laws of falling bodies and the Keplerian laws of planetary motion with his laws of motion, gave birth to the theory of universal gravitation, which encompassed the entire range of the universe then known to man. Immediately after his theory was published, Edmund Halley calculated the orbital period of a comet and predicted the time of its appearance, and it appeared just in the predicted year. The trans-Saturnal planets Uranus, Neptune and Pluto were discovered – each in the each of the successive three centuries after *Principia*. In course of time, therefore, a conviction was born among the scientific community that Newtonian mechanics had said the final words. If there were any discrepancy between the laws of mechanics and a new observation, it was the latter that had be abandoned or modified so as to conform to the laws of classical mechanics.

## Towards a New Relativity

Even then clouds of doubt were gathering in the western sky. As long as physics was concerned solely with matter and linear motion, it was more or less all right. Certain problems of rotation also could be tackled by considering the joint action of two forces acting in two different but linear directions (curvilinear motion). But questions arose as to what happened when a frame of reference rotated with respect to a given inertial frame. Would the laws of mechanics be still valid there, or, if not, how would they be transformed? How did the force of gravitation between bodies separated by great distances act instantaneously? How is it that uniform motion is related to a frame of reference, whereas acceleration, which is also a form of motion of bodies, is detectable in all frames of reference? Some experiments and theoretical developments first in the fields of optics, and then in electricity and magnetism gradually showed dynamic properties which could not be fitted within the purview of mechanics. Ernst Mach in his *History of Classical Mechanics* raised these and other more serious questions concerning the general validity of Newtonian mechanics.[6]

In the field of philosophy there arose a serious problem as regards the source of motion. With Aristotelian physics it was sufficient to refer to force as the source or cause of any motion. Newtonian mechanics had dislodged that role from the shoulder of force. But all bodies of the universe as far as could be visualized were found to exist in motion. Nothing was found to be at rest. Nonetheless, Newton himself admitted the possibility of finding absolute rest in an absolute space. Then what created the first motion? How motion was created from absolute rest? This question had embarrassed Newton too. So he spoke of a "first impulse" exerted by a "prime mover", presumably the omnipotent Provident himself, which once set the world in perpetual motion requiring no further propulsion. The eighteenth century materialist philosophers like Toland[7] in England, Holbach[8] in France and others criticized this weakness of the Newtonian system and strove to brush it aside with the philosophical argument that motion is essential to matter and not extrinsic to it.

Physics itself presented with some new problems. In 1865 James Clerk Maxwell combined the knowledge of the electrical and magnetic phenomena obtained from the experiments of Örsted, Coulomb, Faraday, Lenz, and many others into a set of unified mathematical rules, and immediately afterwards deduced from them the law of electromagnetic wave propagation. It was later found to be broad enough to include all the radiations like light, heat, etc. in its framework as different varieties of electromagnetic waves with different wavelengths and frequencies. Maxwell's equation yielded a finite and constant velocity for any such wave including light in vacuum, which did not depend on the velocity of the source or the observer, given as $1/(e_0.m_0)^{1/2}$, where $e_0$ = permittivity of the medium and $m_0$ = its refractive index. This result is rightly considered a great triumph of theory in the history of physics. Within a short time Heinrich Hertz produced the electromagnetic wave as postulated by Maxwell and confirmed the theory.

This had two serious conflicts with Newtonian mechanics: (a) Velocity of light, or any velocity of the same dimension remains independent of that of the source or medium and does not increase or decrease following the rule $u \pm v$, as is usual for classical mechanics. (b) This velocity of light is the upper limit of velocity attainable by any moving body in empty space.

In such a situation, physics was confronted with two sets of laws, both of which are valid in their respective spheres and experimentally confirmed as true, but which oppose each other. What is the way out?

There was another problem. Already during the lifetime of Newton in the seventeenth century his corpuscular theory of light was challenged by the wave theory propounded by Christian Huygens. Within a few decades of the next century the wave theory of light won the day. Wave propagation required an oscillating medium. So one was postulated – the ether, an invisible, perfectly elastic, all pervading medium occupying all of empty space. Optical phenomena till then known were well explained with this ether hypothesis. Maxwell's equation ultimately raised doubt about the existence of ether. Hertz showed it unnecessary for the propagation of the electromagnetic waves. Albert Michelson in 1881, and Michelson and Morley in 1887 devised experiments to detect ether-drag on the motion of the earth, if any. The result was completely negative for ether. What to do?

Before Einstein, and alongside him, many scientists tried to find a way out. Woldemar Voigt had suggested in 1887 some mathematical forms to show the effects of motion on length and time. Francis Fitzgerald in 1892 postulated some sorts of length contraction and time dilation in the direction of movements of bodies in ether owing to ether-drag and Hendrik Antoon Lorentz developed in 1895 some suitable transformation relations for the changes, which were later usefully absorbed in the work of Einstein. Henri Poincaré produced excellent geometrical analysis of space and time, which helped to remove the idea of ether from physics and he rejected any absolute concepts about space and time. But all of them failed to accomplish the most essential task: how to unify Newtonian mechanical phenomena with the Maxwellian optical cum electromagnetic phenomena into a single system of laws. For they failed to understand that what was required was rejection of the Newtonian concepts of space, time and motion, and a switchover from the Galilean relativity to a new relativity principle.

## Special Theory of Relativity

Einstein dispensed with the ether concept and started from two simple premises: (1) The laws of physics should be equally valid in all sorts of inertial frames of reference; (2) Since therefore Maxwell's laws are also valid in all inertial frames of reference, the velocity of light in vacuum is constant in all such frames. The first premise guided him to seek a new set of transformation laws, which would unify both classical mechanics and electrodynamics. The second told him that no signal could move faster than the velocity of light in empty space; there could be therefore no instantaneous transfer of action. The theory that he built up for the study of dynamical properties of moving bodies on the basis of these two simple postulates is known as the special theory of relativity (STR).

He then raised a simple question: How do we measure the time separation of two distant events? It requires placing two synchronized clocks near the two events and compare the two times of occurrences of the events. And how can we synchronize two clocks? By making them strike the same hours simultaneously. So simultaneity is basic to the measurement of time. The statement of the fact that 'the Howrah-Kalka Mail reached Delhi Junction at 7 PM' is only an abbreviation of the statement that 'the two events – the arrival of the Mail at that station and striking of seven in a station-clock by its hour-hand – were simultaneous'.

Einstein further showed that it is also basic to the measurement of length. Suppose we want to measure the length of a fish swimming in water. We must then place the two legs of a divider simultaneously at the head-end and tail-end of the fish to get its correct measure (see fig. 4). Otherwise, if we place the legs one after another, the measurement will be either longer or shorter than the length of the fish, depending upon the direction of its swimming. All these facts were so obvious to us that – Einstein told – we did not pay any attention to them, nor tried to analyze their meaning.

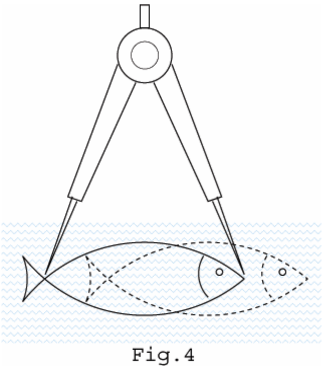

Fig.4

Einstein then posed another question: Is simultaneity of two events absolute or relative? In other words, is it dependent on the frame of reference or not?

In Newtonian mechanics it was tacitly assumed to be the same in all inertial frames of reference. Nobody doubted that it could be otherwise. Since action at a distance was taken for granted, it was also tacitly assumed that any signal could reach any distant object at infinite speed. So an observed simultaneity at a place would also be instantaneously signalized at any distance and be also observed as such. Simultaneity of two events in a place was therefore considered absolute, valid in all places and therefore independent of the frame of reference.

However, Einstein showed, since no signal can be transmitted faster than the velocity of light in empty space, two events occurring simultaneously in one frame of reference may not be so in another frame moving uniformly with respect to it. He clarified it with the help of an easy-to-see thought experiment.

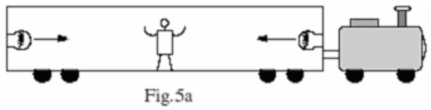

Fig.5a

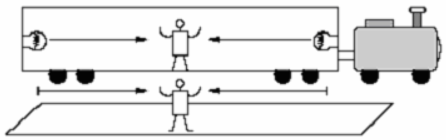

Fig.5b

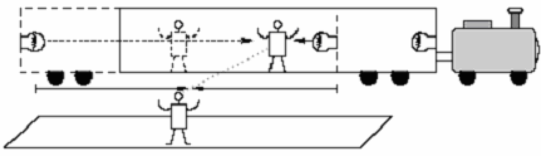

Fig.5c

Suppose, a man standing just in the middle of a long train sees two flashes of light coming from the two ends of the train (see fig. 5a). Since the flashes will reach the middle position at the same time (distance and velocity for both signals being same), the passenger will rightly hold the two flashes to be simultaneous. By virtue of Galilean relativity, remember, the passenger's observation of simultaneity is not at all affected by whether the train is then waiting in a station or moving away from it at a constant speed.

Is it so for another man standing on the platform and viewing the flashes? For him, obviously, the flashes will appear simultaneous if the train is waiting at the station (see fig. 5b). And note that then he and the inside passenger are in the same frame of reference. But if the train is uniformly moving in a straight direction (that is, when the man inside the train and the man on the platform are in two different frames of reference), the platformer will see the front flash to be earlier than the rear flash. For he will see the front flash to have travelled a shorter distance than the rear flash in order to reach the shifting middle position of the train (see fig. 5c). So the two events, which are simultaneous for the passenger in the moving train, are not simultaneous for the man on the platform.

Since our conception and measurement of time (as well as that of length) are based on the idea of simultaneity, which is itself a relative aspect, Einstein argued, measurements of time and length are bound to be different in two different frames of reference moving uniformly from each other. To visualize the difference, again consider a passenger sitting in a train and measuring time by a vertically reflecting light pulse (see fig. 6a). Suppose, the pulse emitted from a source placed on the floor is reflected vertically from a mirror fitted to the inside ceiling of the carriage and back to a receiver on the floor in one unit of time $t$ for the passenger. This will serve him as a quite good clock. Again, note for the n-th time that the passenger will not find any difference in the clock whether the train is at rest or in uniform motion. But what will happen to the train-clock compared to the clock at the station? To see this let us consider what a person with a similar and synchronous clock at the station and standing on the platform see about this clock.

As long as the train is at rest in the station, the platformer will not mark any difference between the two clocks, for he is in the same frame of reference as the passenger. However, when the train is in uniform motion, he will see the clock in the train slow down. Why? For, he will see the signal traverse a longer distance and hence take longer to show the same unit of time (see fig. 6b). The passenger too, on comparing with another clock at the next station synchronous with the previous station's clock will see his clock lagging behind. Similarly, for the same reason, compared to the passenger's clock in the train the clock at the station will slow down. It would not have been so if the velocity of the train could be added to that of the signal, as in the Newtonian system. But the law of electromagnetic wave propagation already forbids that. Also note that the higher the speed of the train compared to the station, the slower the clock of the train as seen from the platform.

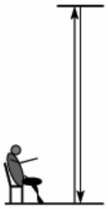

Fig.6a

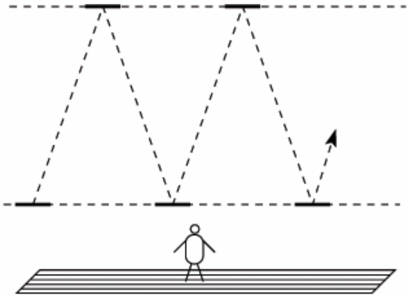

Fig.6b

Let us remember in passing that these are not apparent slowing down of the clocks concerned. The clocks show their own time as real as any. This phenomenon is known in the relativity parlance as *time dilation* due to motion.

Now about length measurement.

We know that length can be measured as a function of velocity and time: $s = v.t.$ Suppose that the train's front passes by the platform in $t$ units of time according to a station clock. In Newtonian mechanics the length of the platform could immediately be calculated from the speed of the train, $v$. But just now we have seen that the clock of the passenger in the running train does not agree with the time shown by the station clock and shows $t'$ units of time to have passed by. Since $t'$ is less than $t$, $s' = vt'$ would also be less than $s = vt$. This means that the man on the platform and the train passenger would not agree on the length of the platform. The passenger would give a shorter quantity than the person on the platform.

Again let us put it in our mind that both these measurements are true and real, they differ only because of the difference of the frames of reference from which they are measured. This phenomenon is described in the theory of relativity as *length contraction* due to motion. This phenomenon is also different from the apparent shortening of length owing to distant vision (as shown in fig. 7), which being a function of the angle and the distance is always same, no matter what an observer sees.

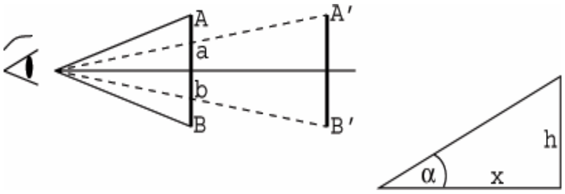

Fig.7: *AB=A′B′* ; but we see *A′B′* as *ab* ; this is a case of apparent shortening.
($h = x \tan\alpha$; with $x$ increasing, $\tan\alpha$ decreeses, the product remains constant).

At this point a question may haunt the reader's mind: If these relativity effects are real, why don't we see them in our day-to-day life? While travelling by the superfast expresses (which in our country sometimes run quite fast) we often synchronize our wristwatches with the station clocks and face no problem. The PWD staff often measure distances on the highways by using the kilometer-readings of a car. And everybody accepts that. How are these possible?

Wait a bit. We are on the verge of entering that point.

## New Transformation Laws

Since the measurements of length and time in the two co-ordinate systems are equally real, there must be some suitable laws of transformation to switch over from one system to the other. Already Voigt, Fitzerald and Lorentz had developed the transformation laws. Einstein also independently derived them in his own method.[9] If the space and time co-ordinates of an event in one frame of reference are given as $x$, $y$, $z$ and $t$, and those in another frame moving along the x-axis uniformly with a velocity $v$ from the first, are $x'$, $y'$, $z'$ and $t'$, then

$$x^2 + y^2 + z^2 = ct^2$$

and $$x'^2 + y'^2 + z'^2 = ct'^2. \qquad (4)$$

By computing the linear relations of the respective terms in these two equations (which is too intricate to be dealt with here), Einstein came to the following four transformation relations:

$$\begin{aligned} x' &= (x - vt) / (1 - v^2/c^2)^{1/2} \\ y' &= y \\ z' &= z \\ t' &= (t - vx/c^2) / (1 - v^2/c^2)^{1/2} \qquad (5) \end{aligned}$$

This set of equations represents the kernel of the STR. Among these, the last one directly gives the time dilation effect. From the first transformation formula we can also easily find out the effect of length contraction owing to velocity. If $x_1$ and $x_2$ represent the end co-ordinates of a line segment moving with a velocity $v$ parallel to the x-axis in a frame of reference, and $x'_1$ and $x'_2$ represent those in another frame moving uniformly with respect to the first with a velocity $v$, then obviously –

$$x'_1 - x'_2 = (x_1 - x_2) / (1 - v^2/c^2)^{1/2}. \qquad (6)$$

In (6), clearly $(x'_1 - x'_2)$ represents the length of the segment in the rest frame (with respect to which its velocity is zero), that is, say, $l_0$, and $(x_1 - x_2)$ represents the length in the frame in which it is moving, that is, say, $l$.

So, $$l = l_0 (1 - v^2/c^2)^{1/2}. \qquad (7)$$

This equation gives for a body moving at a constant speed with respect to an inertial frame of reference the magnitude of contraction in length in the direction of movement. However, Lorentz who had formulated this length contraction before Einstein, regarded this to be due to the pressure of the ether medium against the motion of the body. So he had failed to realize its real physical significance. Einstein for the first time pointed out its significance as a property of space and time.

Now let us see how velocities are to be added in the new system; here a new rule applies:

$$(u + v)/(1 + uv/c^2),$$

where $u$ is the velocity of the body concerned and $v$ is the velocity of the source or the frame of reference;

Let us take two frames of reference $S$ and $S'$ so that $S'$ is moving with respect to $S$ with a velocity $v$. Suppose a train moves in the $S'$-frame with a velocity $u'$, which when viewed from the $S$-frame is $u$. Then

$u$ represents also resultant of the velocities of the $S'$-frame and the train in this frame. In classical physics it would be simply given as: $u = u' + v$.

But here Lorentz transformation will make the case altogether different.

Suppose, $x$ represents the position of the train after time $t$ in $S$-frame, which is $x'$ in the $S'$-frame after time $t'$.

Now, obviously,

$$x' = (x - vt) / (1 - v^2/c^2)^{1/2} = u't',$$

and, $$t' = (t - vx/c^2) / (1 - v^2/c^2)^{1/2}$$

Combining these we get,

$$x - vt = u'(1 - vx/c^2),$$

which can be reordered as

$$x = (u' + v)t / (1 + u'v/t^2) = ut$$

[which is the position of the train in $S$-frame]

Hence, $u = (u' + v) / (1 + u'v/c^2)$. (8)

It is obvious from equation (8) that: (a) when both $u'$ and $v$ are very very less compared to the velocity of light, the term $u'v/c^2$ is virtually zero and $u = u' + v$, as in the classical mechanics; and (b) when either of these two velocities approaches that of light, the resultant is always equal to the velocity of light, that is, $u = c$. This means that the velocity of the train is $c$ in both $S$ and $S'$ frames and is independent of their velocities.

In actual experiment with moving electrons, it has been later observed that in a particle accelerator with a 10 million-volt potential difference an electron attains 99.88 per cent of the velocity of light; when the potential difference is increased to 40 million-volt, the velocity is not doubled, as is expected from the Newtonian relation for kinetic energy and velocity (given as $K_e = 1/2mv^2$), but increases to as much as 99.99 per cent of the velocity of light. Clearly, the velocity of light appears as an asymptote for that of the accelerated electron.

Now we are in a position to answer the readers' question put above. We know, the velocity of light in vacuum is, roughly speaking, 300,000 kilometer per second, or, 10800,00,000 kilometer per hour. In everyday life we experience motion of bodies with velocities very small compared to this. Let us apply the facts in to the STR. As long as the velocity of the body in question, or, of the second frame of reference, $v$, is infinitely small compared to that of light $c$, it is quite evident from the relativity transformation formula that $v/c$ is still smaller and $v^2/c^2$ is quite negligibly small, and can be regarded for all practical purposes as naught. For example, a superfast express which runs at a speed of say 108 km/hour, the ratio $v/c$ is 108/300,000 x 60 x 60 = 0.0001; and hence $v^2/c^2$ is 0.00,000,001. For this kind of velocities the Einsteinian transformations are virtually reduced to the Classical Galilean formula. That is why we do not perceive any effects of time dilation and length contraction in our every day experiences of motion. But for that we were so far wrongly complacent that motion had no effect on length and time measurements. It is only when the velocity of a body approaches that of light that the ratio $v^2/c^2$ becomes significant and the new transformation laws become meaningfully operative.

This further implies that Einstein's relativity is a generalized version of the relativity principle, which contains Galilean relativity as a special or limiting case. The classical Newtonian laws of motion are not rendered wrong, but shown to be of limited applicability in a particular domain.

On the basis of the new theory Einstein showed that space and time are also not independent of each other. For classical mechanics dealing with smaller velocities confronted in our daily life, it was harmless to regard space and time as separate entities independent of each other. But for bodies moving with near-luminal velocities it is quite wrong to do so. At the cosmological level this point can be better understood. When we say that the star Proxima Centauri is 4.2 light-years away from the earth, we also mean that it takes light longer than four years to travel from that star to us. When we look at the star through a telescope, we not only view the distant star but also what it looked like some four years back.

So we view matter in a four-dimensional space-time world picture, in which every point represents an event with four specific co-ordinates – three for space and one for time.

## Mass – Quo Vadis

Interestingly, the STR yielded some more astonishing effects on a moving body till then inconceivable. For example, Let us consider the energy $E$ acquired by a moving object in its displacement from $x_1$ to $x_2$ in one frame and the energy $E_0$ acquired during that from $x'_1$ to $x'_2$ in another frame which is moving with respect to first.

Since the energy gained is force times displacement, the body will have acquired different magnitudes of energy in the two frames of reference, which the lengths of displacements will be different. So, we can write

$$E = F.l \quad \text{and} \quad E = F.l_0$$

From the equation (7), we have

$$E = E_0(1 - v^2/c^2)^{1/2} \qquad (9)$$

Then obviously,

$$E - E_0 = E_0[1 - (1 - v^2/c^2)^{1/2}]$$

Expanding the power term and ignoring the terms containing higher powers than second order, we get

$$\Delta E = ½\,[E_0 / c^2]\, v^2. \qquad (10)$$

If we remember that the LHS of the equation (10) shows the energy difference of the body in going from one frame to another, then the RHS resembles the familiar expression for kinetic energy of a body of (rest) mass $m_0$ and moving with a velocity $v$, that is, $½\, m_0v^2$.

This gives us an equivalence relation between energy and mass of a body :

$$E_0 / c^2 = m_0,$$

Or, $$E_0 = m_0 .c^2,$$

And in general, $$E = m .c^2. \qquad (11)$$

This formula derived by Einstein is very much popular in the world. Those who know or understand nothing of the relativity theory, also know of this.

Since mass is equivalent to energy through a definite quantitative relation, Einstein held, unlike Newtonian mechanics, that a body even when not moving has some rest energy. And when the body gains energy, the extra energy is not solely used up in increasing the speed as presupposed in classical mechanics, but a portion goes into the mass of the body to increase it by a relevant fraction as given by the relation

$$m = m_0 / (1- v^2/c^2)^{1/2} \qquad (12)$$

This explains why the velocity of the electron in the particle accelerator is not doubled when the energy imparted in quadrupled, as already noted above.

Here also it may be noted that when the velocity of the body is far less than that of light, the increase in mass of a body owing to motion is so infinitesimal as cannot be perceived by any measuring device, and, may, therefore, be ignored for all practical purposes.

This new insight into the changing mass of a body owing to motion led to the revision of another set of well established law of classical physics: namely, the laws of conservation of mass and energy. Till Einstein, there were two separate conservation laws; one for the mass and another for energy. Now Einstein pointed out that there should be one single law of conservation of mass and energy together for all bodies of the universe.

In this way Einstein showed that all the fundamental quantities like length, duration, mass, etc., which were so long considered immutable and absolute in the framework of classical mechanics, are actually relative and their magnitudes depend on the states of motion, or the frames of reference in which they move. These have been confirmed in many experiments with the subatomic particles later, when the high energy particle accelerator came into use. Finally Paul Dirac, a great physicist of the century, merged the theory of quantum mechanics with relativity principles and gave birth to what is known as the relativistic quantum mechanics.

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