Gravitational waves: 
Einstein’s whispers from the cosmos

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Ah, gravitational waves, that enigmatic prediction of Einstein’s theory of general relativity. Once purely a source of theoretical debate, they are now humanity’s latest way of learning about the universe. The first success of using gravitational waves as a cosmic messenger came from the Laser Interferometer Gravitational-Wave Observatory (LIGO), which has successfully detected gravitational waves from the coalescence of a binary black hole system [1]. This signal is known as GW150914, since the waves passed through Earth on September 14th, 2015.

The direct detection of gravitational waves is very exciting from the point of view of fundamental physics. However, LIGO is not just a highly sensitive detector. It is an entirely new type of observatory—this binary black hole coalescence is only the first of many that LIGO is expected to detect. These observations, as well as observations of other astronomical events, will give us a wealth of completely new information about the universe: from tests of strong-field gravity to more standard astrophysical matters, like the evolution of binary stars.

In the following article, we will give a brief introduction to gravitational waves, discuss LIGO’s initial discovery, and put it in the context of what we can expect from gravitational wave observations in the future. Further information about GW150914 is available on the LIGO Open Science Centre (LOSC) webpage [2].

History
Any relativistic theory of gravity will predict the existence of gravitational waves—a way to transmit information about changes in the gravitational field from one point in spacetime to another. Einstein first derived the basic properties of gravitational waves in general relativity in 1916. Gravitational waves then became the subject of considerable theoretical debate for the first half of the 20th century. For instance, are they real, physical effects, or just some sort of artefact of the coordinate system one is using? Now the physicality of gravitational waves is a settled matter due to better theoretical understanding and experimental results. Daniel Kennefick’s book gives an excellent account of this history [3].

The first attempt to detect gravitational waves came from Joseph Weber in the 1960s, but with technology (large metal bars) and data analysis techniques that are not as sophisticated as those we now employ. Weber claimed to detect signals, but other researchers were unable to reproduce these results.
The first observational evidence for the existence of gravitational waves came in the late 1970s from radio observations of binary pulsars, where one can precisely measure the masses of the stars and orbital period of the binary by timing the pulsar, as was first done by Joseph Taylor and Joel Weisberg using the pulsar discovered by Taylor and Russell Hulse. One finds excellent agreement with the decrease in the period that general relativity predicts will be caused by the emission of gravitational waves. This measurement has since been joined by a number of similar ones, all of which are in agreement with general relativity.

The 1970s also saw the first proposals for interferometric gravitational-wave detectors like LIGO, which were designed to be able to detect the weak signals one expects from astrophysical sources. The LIGO observatories were inaugurated in 1999. While the initial detectors did not prove to be sensitive enough to detect gravitational waves, this was consistent with astrophysical expectations. In 2015, the first upgrade to the more sensitive Advanced LIGO detectors was completed, and the instruments detected gravitational waves almost immediately.

The physical nature of gravitational waves

Gravitational waves are in many ways a direct analogue of electromagnetic radiation, though the information they can provide about astrophysical sources is more like what we gain from sound in our day-to-day life. They also have some features that are purely their own. Like electromagnetic radiation, they travel at the speed of light and are transverse waves, only acting perpendicular to the direction of propagation. They also carry energy and momentum and have two polarisations, like electromagnetic radiation, but there the similarities end: Fig. 1 shows the two polarisations of a gravitational wave through their effects on a ring of freely-falling test masses. These consist of stretch in one direction with a compensating squeeze in the perpendicular direction (and are thus given the names plus and cross, denoted by + and ×), while the linear polarisation states of electromagnetic radiation move charges back and forth in a line. The magnitude of this squeezing and stretching is proportional to the size of the ring, and is thus measured by the strain

\[ h = \frac{\text{change in separation}}{\text{separation}}. \]

(Note that the strain illustrated here is enormously larger than any gravitational wave strain that could realistically be observed on Earth.)

Additionally, unlike electromagnetic radiation, gravitational waves from even very strong sources are extremely weak—alternatively, one can think of this as implying that spacetime is very stiff: Even

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1Pulsars are rotating neutron stars that emit a beam of radiation that regularly sweeps by the Earth as they rotate. They can rotate very rapidly (up to hundreds of times per second) and are excellent clocks.
very small amplitude gravitational waves carry enormous amounts of energy. For instance, the binary black hole coalescence that created GW150914 emitted the equivalent of 3 solar masses of energy\(^2\), much of it in a fraction of a second during the most dynamical part of its coalescence, leading to an energy flux at Earth greater than that from the full Moon, despite being around a billion light-years away. However, these gravitational waves had a peak strain of \(10^{-21}\), which would correspond to changing the distance between the Sun and the Earth by less than the diameter of an atom.

Gravitational waves also interact very weakly with matter, so they carry information directly from their astrophysical sources to our detectors, without the scattering and absorption that afflicts electromagnetic radiation. However, like sound, they are emitted at wavelengths that are similar to the size of their source, and thus cannot be used to form an image of the source.

The quadrupole formula first written down by Einstein in 1916 gives a reliable guide to the order of magnitude of the gravitational radiation emitted by a given source, even in the strong-field regime—here we give it in the order-of-magnitude form

\[
h \sim \frac{G \text{[mass]} \text{[nonaxisymmetric velocity]}^2}{c^4 \text{[distance to source]}}
\]

(here \(G\) and \(c\) are Newton's gravitational constant and the speed of light, respectively). Note that the amplitude of the radiation falls off as the inverse of the distance to the source and is quite small (at reasonable distances) for even massive and highly relativistic sources, due to the factor of \(G/c^4\). Specifically, if one considers a binary of two \(\sim 30\) solar mass black holes at a distance of \(\sim 1\) billion light-years, orbiting at \(\sim 0.5c\), the speed of the binary that created GW150914 right before its coalescence, the resulting strain at Earth is \(\sim 10^{-21}\), which is indeed the maximum strain observed by LIGO.

How does LIGO work?

Gravitational waves stretch and compress the space between freely falling test particles in a plane perpendicular to their direction of propagation. In an interferometric detector like LIGO, large mirrors serve as these “test particles”—mirrors are placed at the end of two arms at right angles to each other (Fig. 2), and the modulation of the differential optical path length in the two arms is measured by changes in the interference pattern, quite analogous to the iconic Michelson-Morley experiment of the late nineteenth century.

Simple as it might sound, the Advanced LIGO interferometer is able to measure strains of the order of \(10^{-21}\), that is \(10^{-18}\) m of displacement across the arms of length 4 km—a displacement 1000 times tinier than the diameter of a proton. What makes this possible? The simple Michelson interferometer set-up is replaced by a Fabry-Perot cavity in each of the two arms, which acts to amplify the phase shift. Specifically, in these cavities, coherent light from an extremely stable laser bounces back and forth several hundred times as a phase shift builds up. In a ‘lock’ configuration when the instrument is adjusted so there is no light reaching the photodetector when the arm length is unchanged, the interferometer becomes extremely sensitive and can pick up even the tiniest of the displacements we mention.

It is important to remember that whatever signal we see is on top of a background of instrument noise. Instrument noise for advanced LIGO around the time of GW150914

\(^2\)This corresponds to \(\sim 5 \times 10^{47}\) J, using Einstein’s famous relation \(E = mc^2\).
Figure 2: Schematic diagram of a basic Michelson interferometer with Fabry-Perot cavities. The central element is a beamsplitter, a half-silvered mirror that reflects half the light from the laser into the upper cavity and lefts the other half through into the cavity on the right. The black dot at the bottom denotes the photodetector that reads out the light that arrives there if the interferometer’s arm length is changed, shifting it off the dark fringe. Courtesy Caltech/MIT/LIGO Laboratory.

An immense amount of technology and effort goes in to reducing the noise at various frequencies:

- Quantum shot noise (i.e. noise due to counting discrete photons) at high frequencies can be reduced by using higher power lasers, increasing the number of photons.
- Thermal noise at intermediate frequencies is controlled by various means, including using a large beam size to spread out the heating over the mirror and specially designed coatings on the mirror itself.
Mechanical noise at low frequencies is reduced by a series of vibration isolators, essentially oscillators comprising heavy masses and soft suspensions that cut down vibrations at few tens of Hertz at the cost of wider oscillations at even lower frequencies. Additionally, radiation pressure (also at low frequencies) is damped by using heavy (\(\sim 40\) kg) mirrors.

Gravitational wave sources

Compact objects—white dwarfs, neutron stars and black holes—are frequently invoked ingredients in the production of gravitational waves, since they are small enough relative to their mass to be able to withstand the large accelerations necessary to generate appreciable amounts of gravitational radiation. Except for the supermassive black holes found in the centres of galaxies, which may have been formed by some other method, these are the end products of stellar evolution for stars of about the Sun’s mass or heavier (i.e. the remnant left after a star has exhausted all the material it has available to fuse together to resist gravitational collapse).

Stars of about the mass of our Sun to a few times it will end up as white dwarfs, supported by the degeneracy pressure (i.e. the Pauli exclusion principle) of the high density of electrons—while a star expels some of its mass during its evolution to a white dwarf, these objects can have a mass of up to \(\sim 1.4\) times the mass of the Sun, with a radius of about that of the Earth.
Stars with larger initial masses will not lose enough mass to leave a white dwarf remnant. If the remnant is not too massive, it will be a neutron star, supported by repulsive nuclear forces. Here the matter is even more closely packed; the interior of the star is no longer atomic but nuclear matter, a soup of neutrons, protons, electrons and muons (and possibly even more exotic particles) without individual nuclei. These stars can have masses of up to 2 to 3 times the mass of the Sun, with radii of around 10 to 15 km. (The maximum mass of a neutron star and its radius for a given mass both depend on poorly understood nuclear physics at densities above those of nuclei on Earth, but 3 solar masses is quite a firm upper bound for the maximum mass, given well-understood nuclear physics at lower densities.)

Even more massive stars will leave remnants more massive than the maximum mass of a neutron star. These will be black holes, objects of pure spacetime curvature with no material surface, with the matter that formed them collapsed (in classical general relativity) to a singularity. This singularity is cloaked by an event horizon from which nothing, including light, can escape.

Binary black holes are a prominent source of gravitational waves: During the final stages of their coalescence, as emission of gravitational radiation causes the two black holes to approach each other and merge into a single black hole, they are the most luminous gravitational wave sources in the universe. Additionally, since black holes can have large masses (tens of solar masses in the stellar-mass regime, and roughly $10^5$ to $10^{10}$ solar masses in the supermassive regime), they generate strong gravitational waves (the amplitude is proportional to the mass of the system) and can be seen to large distances (billions of light years for ground-based detectors like LIGO, which are sensitive to gravitational waves from binaries of stellar-mass black holes). One can also obtain appreciable gravitational radiation from the coalescence of binaries containing a neutron star, with either another neutron star or a black hole as its companion. In all these cases we only expect to detect signals from sources in other galaxy clusters.

Binaries containing objects that are less compact, even white dwarfs, will have their constituents torn apart by tidal forces (or merge directly at relatively low velocities) before they can generate appreciable gravitational radiation in the audio band accessible to ground-based detectors. However, white dwarf binaries in our galaxy that are not close to merger, with periods on the order of minutes to hours, are a prominent source for proposed space-based detectors, which will be sensitive to gravitational waves in the millihertz band. Such detectors will also be sensitive to gravitational waves from coalescing binaries of supermassive black holes throughout most of the visible universe, as well as binaries of a supermassive black hole orbited by a stellar-mass companion, which will allow for exquisitely precise tests of general relativity, as they will complete millions of orbits in the detector's band, mapping out the spacetime of the large black hole. Heavy supermassive black hole binaries ($10^8$ to $10^{10}$ solar masses) far from merger generate gravitational waves in the nanohertz regime (corresponding to orbital periods on the order of years), which can be probed by timing an array of pulsars.

Compact binaries are not the only prospective gravitational-wave sources. For ground-based detectors, supernova explosions and magnetar outbursts can produce detectable gravitational wave bursts if they...
Another potential source in the Milky Way is a rotating neutron star with a nonaxisymmetric deformation. Such a star would produce a long-lasting periodic signal, but it is unclear if the deformations of any neutron stars are large enough for them to be observable with current or proposed detectors. Additionally, cosmologists are very excited about the prospects for observations of stochastic backgrounds of gravitational waves that could give us information about the very early universe, back to $\sim 10^{-30}$ seconds after the Big Bang.

However, there are also stochastic backgrounds formed by e.g. all the compact binaries in the universe, and these may obscure any cosmological backgrounds (but nevertheless carry interesting information themselves). See Fig. 4 for an overview of the noncosmological gravitational wave spectrum and the position of various detectors on it.
Source modelling

In order to detect gravitational waves signals that are buried in noise, and to infer the parameters of the system that emitted the waveform, one needs to be able to quickly generate highly accurate template waveforms for all possible astrophysical sources. Fortunately, for coalescences of binary black holes, the waveforms are relatively simple, and can be computed to high accuracy using only Einstein’s equations of general relativity. Of course, this is much easier said than done, and computing these waveforms has been the subject of a number of researchers’ entire careers.

One can compute analytically, by expanding Einstein’s equations in a slow-motion and weak field approximation (known as the post-Newtonian approximation). However, while these analytic approximations generate waveforms quickly and also provide considerable intuition about general relativity’s predictions for the motion of compact binaries, they cannot describe the very dynamical merger phase of the binary. Fortunately, it is also possible to solve Einstein’s equations on a supercomputer with no approximations, besides the discretization necessary to make the equations amenable to numerical solution, whose associated error can (in principle) be made arbitrarily small. While such solutions are now relatively routine, the breakthrough that made them possible only occurred in 2005 after significant technical and conceptual developments.

However, numerical solutions to Einstein’s equations are quite computationally intensive, often taking weeks or longer of supercomputer time to compute just the last 5 or 10 orbits of a given binary. Researchers have thus developed fast-to-evaluate models for the waveforms that include the results from analytical approximations, which can accurately describe the early stages of the inspiral, and are calibrated with the numerical solutions in the final stages of the evolution near merger.

Search for the signal and estimation of parameters

The signal is buried in noise, often with amplitude larger than that of the signal itself. However, unlike the random, incoherent noise, the signal is a coherent pattern that can be accurately modelled. This modelled signal can be searched for in the data—one can scan through the data looking for signals expected from various model parameters—in case a signal is present in the data, it shows up as a peak in the cross correlation around the values of the signal parameters. This technique is known as “matched filtering”. Since one uses template waveforms for this search, it is crucial to model the waveforms accurately, as discussed above.

Following a fast “search” that gives a crude idea of the waveform parameters, one needs to perform rigorous parameter estimation. The parameters to estimate include the intrinsic parameters of the system—the masses and spins, along with additional deformation parameters for neutron stars—and the extrinsic parameters—the distance to the system, its location and orientation in the sky. With a large number of parameters to estimate, one resorts to a stochastic sampling of the probability distribution on the space of parameters.

Estimation of parameters is crucial for various reasons. A quick estimation of the location in the sky is required to alert observational astronomers to perform follow-up searches. The more rigorous estimation of parameters eventually obtained can tell us about consistency of the waveform with predictions from general relativity, the physics of matter within neutron stars, the viability of astrophysical models and
predicted rates of mergers, and even cosmological parameters.

**GW150914 Results and Implications**

On September 14, 2015 at 09:51 UTC (3:21pm IST), the LIGO detectors at Livingston and Hanford detected a strong gravitational wave signal with a signal-to-noise ratio of 24. The strength of this signal allowed us to independently detect it in multiple search pipelines which depend on very different algorithms, both generic unmodeled waveform searches and optimal searches using matched filtering. The former searches do not make any assumption about the validity of general relativity or the nature of the source, while the latter utilise very accurate source modelling in general relativity. The matched-filter analysis indicated that the signal was consistent with a merger of a binary of two black holes of masses of about 30 times the mass of the Sun. The probability that noise alone could mimic such a signal is less than one in 7 million. A careful analysis showed that the observed gravitational wave is from the last 0.2 seconds of the merger of black holes of component masses of about 36 and 29 times the mass of the Sun, at a distance of about 410 megaparsecs (1.3 billion light years). The event is localized to a patch on the sky of area 600 square degrees, mainly over the Southern Hemisphere. See Fig. 5 for an illustration of the detected waveform and a theoretical template consistent with it.

Although the spins of the initial component black holes are not very well estimated (only can only say the heavier black hole did not have more than 0.7 times the maximal allowed spin), the spin of the final remnant black hole is quite well-estimated to 0.67 the maximal spin allowed for rotating black holes. This is one of the most accurate estimations of the spin angular momentum of a black hole. The mass of the final remnant black hole is 62 solar masses. Thus, a total energy of about 3 solar masses was radiated as gravitational waves, mostly in a fraction of second around the merger—the peak luminosity of the radiation is estimated to $3.6 \times 10^{51}$ erg/s, which is roughly 100 times brighter than the luminosity one would infer for the most luminous gamma-ray burst if it emitted its energy isotropically, instead of being strongly beamed, as it is expected to be.

The observed signal is found to be consistent with a binary black hole merger as predicted by general relativity. When the best-fit general relativity waveform is subtracted out from the signal, the residual data is completely consistent with noise at other times when no signal is present. The mass and the spin angular momentum of the final remnant black hole, estimated independently from the early “inspiral” and the late “merger-ringdown” stages are found to be consistent with each other, given the expectations from general relativity. There are no observed departures from the analytical waveform models obtained from general relativity, and much stronger constraints than previous ones are laid on the departures from general relativistic values of the post-Newtonian coefficients that parametrize the waveform models. The graviton field, expected to be massless, is demonstrated to have a mass consistent with zero, and stronger bounds than before are placed on this mass from the observation that there is no dispersion during the propagation of gravitational waves (i.e. gravitational waves of different frequencies all travel at the same speed).

Just after the merger, the newly-formed black hole is in an excited state and will radiate away the energy and angular momentum in the perturbations with some char-
Figure 5: The data in the two interferometers and a theoretical template (from a large-scale binary black hole supercomputer simulation) that is consistent with the observed waveform, all obtained from the LOSC [2]; compare Fig. 1 in [1]. The upper two panels show the data and template after filtering by a 30–350 Hz bandpass filter, to concentrate on the detector’s most sensitive region, and further filtering to remove the various instrumental lines seen in Fig. 3. Such filtering is done solely for the purposes of this figure to make the signal stand out from the instrumental noise—it is not used in our analysis. One can see the ~7 millisecond time shift between the signals arrival at the Livingston and Hanford detectors. The difference in amplitude between the two detectors is due to their differing orientation with respect to the source. All times are shown relative to September 14th, 2015, 09:50:45 UTC. The bottom panel shows the template waveform as it would appear in the Hanford detector with no filtering.

The measured frequency and damping time of this least damped “quasinormal” mode is found to be consistent with the theoretical expectation for a black hole with the final mass and spin we infer from the data. All these tests are some of the first tests of general relativity in the strong-field regime, where the velocities of the objects being considered are almost half that of the speed of light.

This event reveals that binary stellar-mass black holes form in nature and merge within the age of the universe. This observation also reveals the existence of stellar-mass black holes more massive than the 25 solar masses previously inferred from electromagnetic observations. Formation of such massive black holes from stellar collapse implies that the stars were formed in an environment without many of the heavier elements and had a weak stellar wind (see e.g. [6] for discussion of a potential formation channel).

From this observation, LIGO has been able to estimate the rate of stellar-mass binary black hole mergers in the local universe to be 2–400 per cubic gigaparsec per year in the comoving frame (1 gigaparsec is about 3 billion light years) [7]. This is consistent with earlier predictions, though
Towards the higher end. Detection of a few tens of binary black hole mergers will allow us to understand the stellar evolution of massive binary stars in galactic fields and possibly also probe stellar interactions in dense regions such as globular clusters.

This observation implies that the stochastic gravitational-wave background from binary black holes, created by the incoherent superposition of all the merging binaries in the Universe, could be higher than previously expected. This background is potentially measurable by the Advanced LIGO/Virgo detectors operating at their projected sensitivity.

**LIGO-India**

Current plans call for a world-wide network of ground-based gravitational wave detectors that will become fully functional starting in the early 2020s. The addition of more detectors will improve our sensitivity to gravitational waves and also reduce the impact of downtime in any one detector. However, the most important improvement will be in our ability to locate the source on the sky: For gravitational wave detectors, such localization largely relies on timing—the difference in signal arrival time between the detectors lets one measure the direction from which the signal is coming—longer distances are thus a big help here. The upgraded Advanced Virgo detector in Italy is expected to become operational in 2016. The KAGRA detector in Japan is in the process of construction and is expected to become operational around 2018. The LIGO-India project, which recently received in-principle approval from the Indian cabinet, is a proposal in which a third LIGO detector will be built and operated in India, in collaboration with LIGO-USA and its international partners. India is in an excellent position for a detector geographically, with almost the maximum possible distance from the detectors in the US. LIGO-India will thus be a critical element in allowing the network of detectors to do astronomy, particularly in engaging with traditional electromagnetic astronomy to follow up sources.

The LIGO-India project will be led by teams at the three lead institutions, the Institute for Plasma Research, the Raja Ramanna Centre for Advanced Technology and the Inter-University Centre for Astronomy and Astrophysics, partnered with the LIGO Laboratories in the US. Initial site selection has already started—the detector will likely be located somewhere on the seismically stable Deccan Plateau.

**What next?**

LIGO is currently undergoing another upgrade to further improve its sensitivity. It will start its next observing run in mid-to-late 2016, for which it will be joined by the recently upgraded Advanced Virgo detector [8]. This run is expected to last for about six months, and we can expect to detect at least a few more binary black hole coalescences during that time, based on the rates of binary black hole coalescences we infer from our observation of GW150914 [7]. We can also hope to observe a compact binary coalescence containing a neutron star or gravitational waves from an isolated neutron star—possibly even something completely unexpected. Any of these would be another major first for LIGO and Virgo.

In subsequent years, the LIGO detectors and Virgo will undergo further upgrades to higher sensitivities, with Advanced LIGO predicted to reach its design sensitivity around 2019. They will also be joined for joint observing runs by the Japanese detector KAGRA, and eventually by LIGO-India. Additionally, pulsar timing observations
will continue and increase in sensitivity. They have already placed constraints that rule out several models for the evolution of supermassive black hole binaries [9], and can potentially make a detection of a heavy supermassive black hole binary or the background due to a population of them in the future.

Looking further into the future, there are plans for even more sensitive ground-based detectors, notably the Einstein Telescope, which would be able to detect binary neutron stars 100 times further than Advanced LIGO at its design sensitivity [10]. Additionally, there are plans to build a detector in space, eLISA, which will be sensitive to millihertz gravitational waves [11]. The LISA Pathfinder experiment that was launched in 2015 has now started testing the technology crucial for eLISA to detect gravitational waves.

We thus look forward to learning much about the universe via gravitational waves, both from expected sources, as well as potentially things we never even thought of. While it might seem appropriate to say that the future of gravitational wave astronomy is bright, that is a very electromagnetic way of putting things. Thus, we will end by saying that the future of gravitational waves is (relatively) loud/strong!

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References