A Brief History of Science
Part 13: The Development of Quantum Mechanics

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Introduction

BY THE 1890s, the pillars of physics — the Newtonian theory of gravity and dynamics that explained the motion of bodies, Maxwell's theory of electromagnetism that explained all electrical and magnetic phenomena including the nature of light, and thermodynamics which explained the phenomena resulting from exchange of heat — were on firm footing. Physicists grew complacent and believed that there was nothing more to be done in this field.

"All that remains is to dot a few i's and cross a few t's", commented the physicist John Trowbridge of Harvard University. Albert Michelson of Chicago University (a future Nobel Laureate) said in a lecture, "The future truths of physics are to be looked for in the sixth place of decimals."

Yet in the next 30 years physics saw a revolution that completely changed our perception of the material world. Two path-breaking theories made their appearance— the theory of relativity and quantum mechanics. This happened in the intellectual atmosphere of struggle between positivism and materialism. In this instalment we shall discuss the history of development of quantum mechanics.

The realization that the knowledge of the time cannot explain the material world came mainly with two discoveries: black-body radiation and radioactivity. We have discussed the former in the last issue. So let us start with radioactivity on our way to quantum mechanics.

Radioactivity

In 1895, Wilhelm Conrad Roentgen discovered the x-ray, and captured the image of the bones of his hand on photographic plates. To publicize the event, he mailed the photographs to a few eminent scientists. That created a sensation, and within three weeks the new technique of x-ray was being used by medical practitioners to set broken bones.

Becquerel knew about the phenomenon of phosphorescence, that some materials glow in the dark after absorbing energy from the sun during daytime. He guessed that in phosphorescence, the materials emitted x-rays. As a test, he exposed a phosphorescent material, potassium uranyl sulphate, in the sun, and then covered it with black paper and put it on a photographic plate. The plate, when developed, turned black. He thought that this material was emitting x-rays just like Roentgen's rays.

The next few days were cloudy, and so Becquerel put the whole contraption in his drawer. A week later, when the

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sun came back, he intended to resume his experiment. But instead of putting the potassium uranyl sulphate out in the sun, he first wanted to test how good the photographic plates were, and so he developed one of them. Surprise: It turned black, meaning that it has been exposed to radiation even though the material had not absorbed sunlight. The discovery was serendipitous, but the importance of such chance factors can be grasped only by a trained mind. Becquerel did systematic investigation for a few months and established that the material was emitting the rays all by itself. He showed that the rays contain energy—because substances that absorbed the rays became heated. He showed that dry air, which normally does not conduct electricity, became conducting in presence of these rays—the extent of which can be measured with electroscopes.

At this point of time, Marie Sklodowska Curie, a young student from Poland, was looking for a suitable problem to do her research. Becquerel's discovery attracted her attention as it posed a few questions.

I shall discuss Madame Curie's method of investigation in some detail because today's research students can learn many things from it regarding the method of scientific research. Any scientific research starts with a question. So she asked the question: Is radioactivity a property of a compound or of an element? To seek answer to this question, she prepared a few compounds of the same mass, but in which the quantity of uranium was different. By measuring with an electroscope, she found that radioactivity in these compounds were different, but was proportional to the amount of uranium in each compound. Thus she concluded that radioactivity is a property of the element uranium.

Then she asked the question: Is radioactivity a property of only the element uranium, or do other elements have the same property? She took various compounds that contain different elements and measured the radioactivity of each. This way she examined all the elements discovered till that time, and found that another element, thorium, is also radioactive.

Then she argued that the minerals found in nature should exhibit radioactivity if they contain uranium and thorium, and should not exhibit radioactivity if they don't contain these two elements. She examined hundreds of minerals and checked that the hypothesis was indeed true. Then she did something strange: she argued that in the minerals that contain uranium and thorium, the radioactivity due to these two elements individually should add up to give the radioactivity of the mineral. So she measured the quantities of uranium and thorium in these minerals, and checked if the radioactivity of the mineral is a simple sum of the radioactivity of the uranium and thorium present in the mineral. She found that this is true for most minerals, but in the mineral called pitchblende she found that its radioactivity exceeds that expected by considering its uranium and thorium contents individually.

She hypothesized that pitchblende contains a hitherto unknown element that is highly radioactive. Why was it not detected in her chemical analysis? That is because it occurs in minute quantities. She needed to isolate this substance. Since her research opened such an exciting possibility, her husband Pierre joined her pursuit. They painstakingly isolated each element in the mineral, and measured the radioactivity of each. To their surprise they found that not one but two elements that are known to be non-radioactive—bismuth and barium—are exhibiting radioactivity. They realized that pitchblende contains not one but two new radioactive elements, the chemical

Pierre Curie (1859-1906)
and Madame Curie (1867-1934).

properties of the first are similar to that of bismuth and the properties of the second are similar to that of barium. They named one as 'polonium' and the other 'radium'. The reader may note how Madame Curie made use of John Stuart Mill's ideas on operational causality (see Part-9 of this essay) in designing her experiments to answer the question "what causes radioactivity"?

The story does not end there. They now faced the task of isolating these two new elements and measuring their atomic weights. It was a herculean task since these elements occur in trace quantities. So they would need an enormous quantity of pitchblende, which is an expensive mineral. How would they get such money?

They found a solution: there were companies that extracted uranium and thorium from pitchblende and threw away the remaining material. They realized that the elements they were looking for must be contained in this leftover substance.

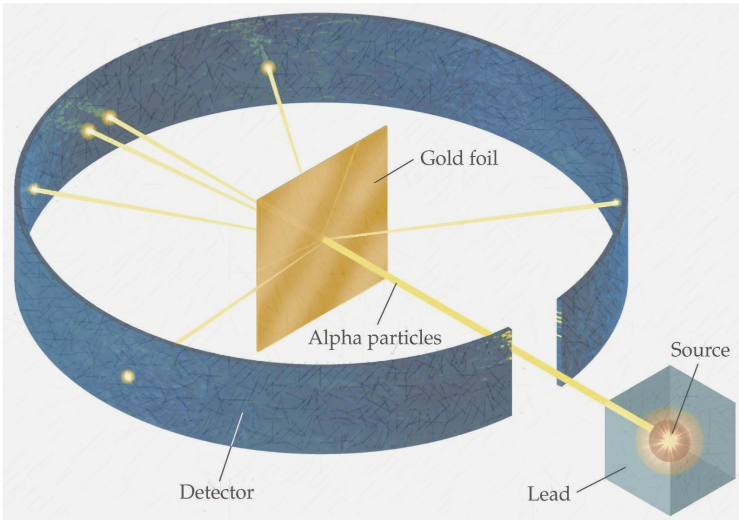
So they arranged to get a thousand kilograms of this material, and started the painstakingly laborious process of processing one kilogram each day to isolate bismuth and barium from the substance, and subject these to further processing to extract the two new elements. They were finally able to isolate polonium and radium in sufficient quantities to measure their atomic masses. Along with Henri Becquerel, they were awarded the Nobel Prize in 1903.

Radium turned out to be highly radioactive—its radioactivity is a million times that of uranium. And so it turned out to be the ideal source if one wanted to experiment on radioactivity. It is also extremely rare. By 1916 the world's store of radium was less than half an ounce. But Madame Curie parcelled out small amounts of the new element to whoever wanted to experiment on it.

Looking inside the atom

When J J Thomson discovered the electron through the study of cathode rays, there was considerable reluctance in the scientific community to accept it. "It is difficult to grasp how startling the notion of a subatomic particle was to the nineteenth century physicists, many of whom did not believe that atoms existed, let alone they had constituent parts" [1]. But the study of radioactivity was increasingly revealing that there must be things inside the atom. It was found that the beta rays emitted by radioactive substance were nothing but electrons. If there are negatively charged electrons inside the atoms, there must be something positively charged also, because the atoms are neutral.

Ernst Rutherford, sitting in distant McGill University in Canada, obtained a bit of radium generously parcelled by Curie, and proceeded to investigate the character of alpha rays. He measured the



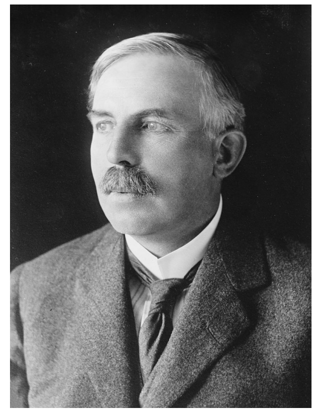


Illustration of Rutherford, Geiger, and Marsden's experiment and Ernest Rutherford (1871-1937).

charge-to-mass ratio of the alpha particles and concluded that these were positively charged helium atoms. But he noticed that observations of the alpha particles are extremely difficult because the particles are constantly scattered by everything in the laboratory, including air. Unable to reduce the scattering in spite of repeated attempts, he decided to focus on scattering itself. By then, he had moved to Manchester in England. In 1908, he asked his students Hans Geiger and Ernst Marsden to observe the scattering when alpha particles fell on a thin metal foil.

By that time, in spite of the positivists' objections, people had started to believe in the existence of atoms (after Einstein's arguments, see part 12 of this article), and the discovery of radioactivity indicated that atoms were made of smaller constituents. The mental picture of the time was that negatively charged electrons were embedded in a ball of positively charged substance, much as pieces of fruit are embedded in a fruit-cake. It was called the "plum-pudding" model.

Geiger and Marsden's experiment revealed that the alpha particles are scattered mostly around the direction of travel, but one in eight thousand particles would rebound right back in the opposite direction. This puzzled Rutherford, because the result was not expected from the existing plum-pudding model of the atom. After many repetitions of the experiment and much groping in the dark, around 1911 he found the answer: the observation indicates that the positive charge of the atom is concentrated in a tiny blob at the centre of the atom, while the electrons are whizzing around this 'nucleus'. This gave rise to a solar system like mental picture of the atom.

Very few physicists paid attention to Rutherford's proposition regarding the structure of the atom, because it had a serious flaw: according to the theory of electromagnetism, an electron going round a nucleus (which is an accelerated motion) would continuously radiate energy and would drop into the nucleus in a fraction of a second. The 'solar system' like atom,

Niels Bohr (1885-1962)

therefore, cannot be stable. Yet we do see stable matter all around us!

In 1911, a twenty five year old Danish physicist named Niels Bohr was visiting Thomson's Cavendish laboratory in Cambridge and Rutherford's laboratory in Manchester. There he heard of Rutherford's idea about the structure of an atom, and took it seriously. Earlier the idea of light quanta (the fact that light comes not as a continuous stream but as discrete 'packets' of energy) had been proposed by Planck in 1900, and Einstein had demonstrated in 1905 that light is emitted and absorbed in similar 'packets' (see Part 12 of this article). Bohr guessed that the light quanta as proposed by Planck and Einstein were in some way responsible for the stability of atoms.

He postulated that electrons in atoms can move only in certain stable orbits and can jump from one to another by absorption or release of a quantum of energy. Assuming circular orbits of the electron, the laws of classical mechanics and the above 'quantum' postulate, Bohr managed to show that absorption and emission of light by hydrogen can happen only at certain frequencies. It was known that the spectrum of light passing through hydrogen shows a few dark lines corresponding to the frequencies that are absorbed by hydrogen, and the mathematician Balmer had given a formula for these frequencies. Bohr's prediction exactly matched Balmer's formula, and by that, his postulate explained why the Balmer lines occur in the hydrogen spectrum.

The paper published in 1913 caused quite a stir, because, for the first time scientists had an explanation of the spectral lines. But it was soon found that, while Bohr's theory obtained the correct values of the frequencies of the spectral lines of hydrogen, it shed no light on the intensities of these spectral lines. Moreover experimentalists found that there were faint lines around the major spectral lines, and Bohr's theory provided no explanation for that. Arnold Sommerfeld (1868-1951) tried to overcome this weakness by assuming elliptical orbits, but it was soon found that the Bohr-Sommerfeld line of approach cannot predict the spectral lines of anything other than hydrogen—the simplest atom.

Physicists groped in the dark for quite some time, trying to reconcile the well-known laws of classical physics and electromagnetism with the new experimental findings of atomic phenomena, but with no success. Then from 1924, things began to move really fast.

The solution

Earlier Einstein had demonstrated that light, which is known to be of wave character (recall interference and diffraction), also has a particle character. In 1924 Louis de Broglie postulated, in a similar vein, that what were known as particles (electrons, protons, etc.) also have a wave character. G P Thomson (son of J J) in England and

Davisson and Germer in the United States experimentally demonstrated in 1927 that streams of electrons passing over obstacles exhibit diffraction pattern, a sure sign of wave character. But waves of what? That question was yet unresolved.

In July 1925, the young Werner Heisenberg took a shot at the hydrogen spectrum in a different way. Bohr had postulated that absorption or emission of radiation from an atom can happen only when an electron jumps from one level to another. Assigning numbers 1, 2, 3 $\cdots$ to the different levels, Heisenberg arranged the possible transitions and the associated energies and other variables in square arrays and proceeded to manipulate such arrays to obtain useful results. Now we know that such arrays are matrices, but in the 1920s matrices were unknown to physicists. Max Born first realized that these are matrices, and in a paper that appeared in September 1925, he and his student Pascual Jordan proceeded to apply the rules of manipulation of matrices that were given by mathematicians.

They soon realized that there was a problem: while two ordinary numbers can be multiplied in any order and the result is always the same (i.e., $a \cdot b = b \cdot a$), in the multiplication of matrices the order matters. So if these matrices in some way represent the physics of the micro-world, that physics would be quite different from the physics of classical mechanics and electromagnetism. In particular, they found that if the position q and momentum p of a particle are represented by such matrices, then $p \cdot q$ is not equal to $q \cdot p$, that is, in modern language, these two variables do not commute. This result had a far-reaching implication that was to be revealed later.

While Bohr, Heisenberg, Jordan, and others were working out this 'matrix mechanics', in 1926 Erwin Schrödinger approached the problem from a completely different direction. It was known that the common perception of 'ray of light' is only a mental construct: the line perpendicular to the propagating wave front. For long-wavelength radio waves, such 'rays' lose meaning the way we do not talk about 'rays' of sound. Thus the straight line propagation of light is only a consequence of its wave nature. Schrödinger added to this de Broglie's assertion that particles also have wave character. He guessed that the propagation of a particle could, in some way, be explained by the evolution of its associated wave, and proceeded to construct a theory of micro-particles based on the well-known theory of waves.

Louis de Broglie (1892-1987)

In fact, another clue led him in this direction. In 1924, half the globe away from the centre of activity, in the University of Dacca (now in Bangladesh), Professor Satyendra Nath Bose was teaching Planck's derivation of the black body radiation curve to his students. He did not like Planck's approach and derived it on his own. He then wrote up his derivation as a paper and sent it to Einstein with a request that, if he approved it, he may kindly get the manuscript translated into German and get it published. Einstein realized that Bose had made a major discovery, and promptly got it published.

In his derivation, Bose had made a daring assumption that photons are indistinguishable from each other, and had built his statistics on that basis. Einstein then went a step further and assumed a 'gas' composed of particles that are indistinguishable. In a paper published in 1925, He showed that such a gas would undergo a qualitative change in character at low temperatures—a phenomenon now known as 'Bose-Einstein Condensate' [1]. Using this knowledge, Schrödinger argued that, if light behaved as waves as well as particles, and if Einstein could use the same kind of statistics to atoms that actually apply to photons, why not take it a step further and try to construct a wave theory of particles?

He assumed that there is a quantity, denoted by the Greek character Ψ (psi), that embodies this wave character, i.e., goes up and down in a wavelike manner. He wrote down an equation that captures this variation of Ψ (called the Schrödinger equation). When he applied the equation to a particle bound by some kind of force (say, an electron tied to an atom) and solved the differential equation, the discrete or 'quantized' values of energy emerged as a natural consequence of the wave nature—in a way similar to the common observation that strings tied at the two ends can produce only certain notes, i.e., discrete values of frequency. And for the hydrogen atom, the discrete energy values that emerged matched exactly the observed ones.

Thus, there was a peculiar situation: two different mathematical formalisms—Heisenberg's matrix mechanics and Schrödinger's wave mechanics—seemed to account for the observed facts, both successful in their own ways. Finally, Schrödinger solved the quandary by showing that these two approaches are in fact equivalent, two sides of the same coin. One could use either of them to arrive at the correct answers. Slowly scientists found, through their practice, that Schrödinger's method is easier to use, and now science has practically forgotten Heisenberg's matrix mechanics.

Werner Karl Heisenberg (1901-1976)

But what was this Ψ? What is its physical interpretation? Schrödinger took a shot at this question, but his answer turned out to be incorrect. Then Max Born showed that Ψ is related to probability—the probability that the particle exists in a certain location is given by the square of the magnitude of Ψ evaluated at that location. Thus the statistical interpretation of quantum mechanics was born.

The statistical interpretation said that it is impossible to pinpoint the position of a

[1]The Bose-Einstein condensate was experimentally observed by Eric Cornell, Carl Wieman, and co-workers on 5 June 1995.

particle and we can only get a probabilistic estimate of where it could be. It has different probabilities of being in different positions, which is given by the wave function Ψ. The evolution of the wave function, in turn, is governed by the Schrödinger equation. It turned out that the momentum of a particle also has to be specified in probabilistic terms, i.e., we cannot say what the momentum of a particle exactly is, but we can calculate the probability of having a momentum lying between two specific values.

Then Heisenberg showed in March 1927 that the 'spreads' in the probability distributions of position and momentum cannot both be arbitrarily small. If the standard deviations of the distributions of position and momentum be Δx and Δp respectively, then $\Delta x \cdot \Delta p$ should be greater than $h/4\pi$, where h is the Planck's constant. This is the celebrated uncertainty principle of Heisenberg. Note that this is a mathematical result, not a result of our attempts to observe the position and momentum of a particle, nor is it a matter of efficacy of our instruments. In fact, all the pairs of variables that do not commute (for example, the angular momenta in the x and y directions) have this property.

All these developments happened over a brief period from 1924 to 1927. The basic formalism of quantum mechanics was laid out within these three years. After this period, major contributions were made by Wolfgang Pauli (the exclusion principle), Paul Dirac (quantum electrodynamics) and many others that opened up the new branch of particle physics.

But the development of quantum mechanics created intense controversy regarding its interpretation and philosophical implication. Let us now turn our attention to that.

Max Born (1882-1970)

The controversies

There were basically two central issues on which the scientists of the time could not agree with each other. The first concerned the probabilistic nature of reality. Classical physics was based on strict determinism: a given initial condition of a body necessarily leads to a specific final state after a lapse of time, and classical physics provided the tools by which the evolution from the initial state to the final state could be exactly calculated. In contrast, quantum mechanics enabled one to calculate only the probabilities of various possible outcomes starting from a given initial state. Some scientists, including Einstein, contended that this probabilistic description is on account of our ignorance of the exact position and momentum, and there is a fundamental reality behind the quantum probabilities, which quantum mechanics has not grasped. When the missing pieces are assembled, the probabilistic nature will disappear and we'll again have a deterministic description of microscopic phenomena.

A conference, called the Fifth Solvay

Box-1: The postulates of quantum mechanics
(for the mathematically inclined reader)

- The state of a particle is given by the wave function $\Psi(x,t)$ which has different values at different points in space, and varies with time. Ψ is in general a complex number.
- The probability of finding the particle in the range between a to b is given by $\int_a^b |\Psi(x,t)|^2 dx$. Since the particle must be *somewhere*, $\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$.
- When a particle of mass m is subjected to a potential function $V(x)$, the wavefunction evolves deterministically according to the Schrödinger equation

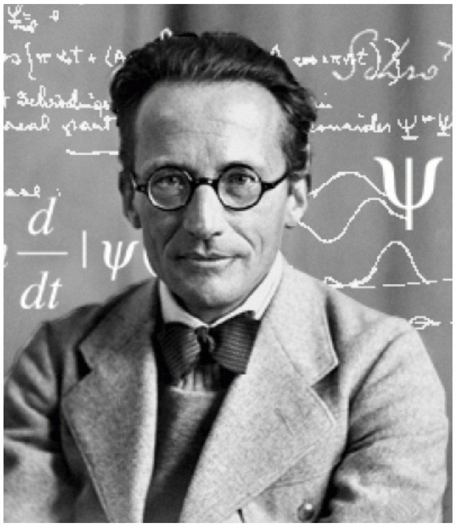
$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t) \quad \text{where } \hbar = h/2\pi.$$

- We cannot observe the state of the system, but can measure the 'observables'. The observables are given by *operators*. For example, 'multiplication by x' is the operator for position, $-i\hbar\frac{\partial}{\partial x}$ is the operator momentum, etc.
- Every measurement of an observable yields one of the eigenvalues of the corresponding operator.
- If the operator corresponding to an observable (say, energy) has n eigenvalues, we cannot say which energy value will be observed in a measurement. But we can state the probability of observing the i-th eigenvalue, which is given by

$$p_i = \left| \int_{-\infty}^{\infty} \psi_i^*(x)\Psi(x,t)dx \right|^2,$$

  where ψ_i is the corresponding eigenfunction.
- A measurement causes the wave function to jump discontinuously to an eigenstate of the dynamical variable that is being observed.
- The average or 'expectation' value of an observable (represented by the operator $\hat{O}$) is given by $\int_{-\infty}^{\infty} \Psi^* \hat{O} \Psi dx$.
- If $\hat{A}$ and $\hat{B}$ are two operators that commute, i.e., $\hat{A}\hat{B} - \hat{B}\hat{A} = 0$, then the corresponding variables can be simultaneously measured with infinite precision. But if they do not commute, the corresponding variables cannot both have precise values at any point of time.

International Conference on Electrons and Photons was convened in October 1927 at Brussels, where the world's most notable physicists met to discuss the newly formulated quantum theory. In this conference, Einstein raised a few objections about the statistical interpretation, especially about the uncertainty principle, in his characteristic style—by proposing thought experiments and demonstrating that these would lead to contradictions. Scientists would spend sleepless night trying to find answers to the questions Einstein had raised, and the next morning Bohr would come forth with the appropriate logic to show that there would be no contradiction. The same thing continued in the Sixth Solvay Conference in 1930. Finally Einstein and other opponents (like Planck, de Broglie, etc.) conceded that quantum theory is a correct theory, at least as far as its mathematical methodology is concerned.



Erwin Schrödinger (1887-1961)

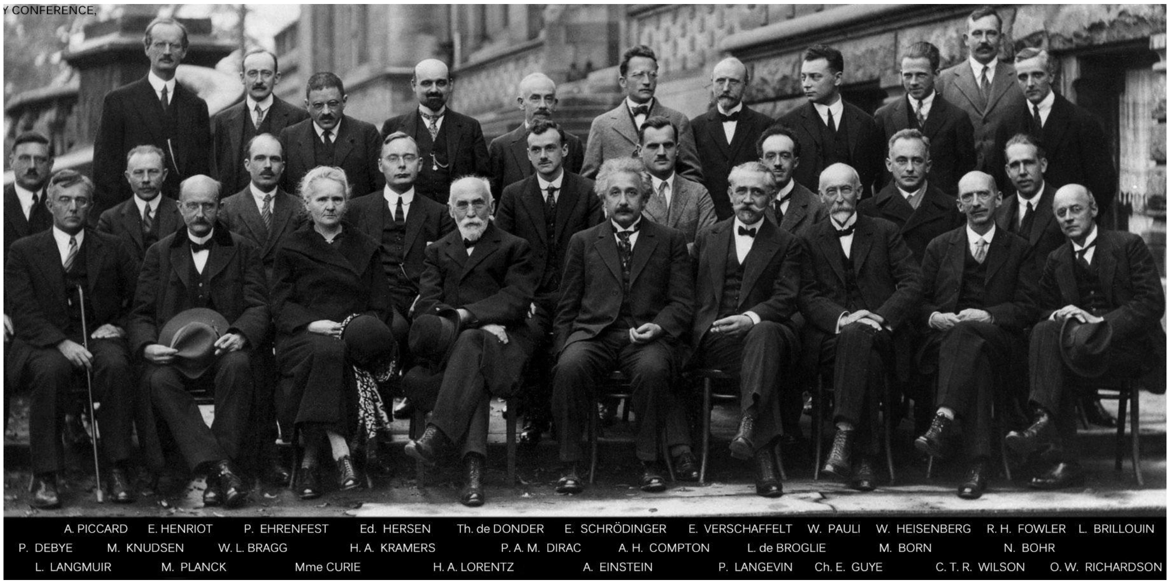
But Einstein never changed his second point of objection. What was it?

In order to understand the nature of the controversy, we need to see how the theoretical structure of quantum mechanics was interpreted. The most prevalent one is known as the Copenhagen interpretation, which was developed mainly by Niels Bohr and Werner Heisenberg. It contended that the real character of the micro-world is not amenable to experimental investigation because any attempt to observe it will invariably disturb what we are trying to observe. Therefore, we should abandon all attempts to know the character of physical reality and instead should only focus on what are observable. They went a step further and said that physical reality does not exist until we observe it. In the language of Heisenberg, "Atoms or elementary particles are not real; they form a world of potentialities or possibilities rather than one of things or facts." When an observation is made, say, on a particle's position, we force the particle to take a decision: out of the many possible positions, one actualizes. In the language of the physicist Pascual Jordan, "Observations not only disturb what has to be measured, they produce it ⋯ We compel [the electron] to assume a definite position ⋯ We ourselves produce the results of measurements." Thus, according to them, conscious intervention creates reality. There is no reality existing independent of our consciousness. It is clear that this line of argument comes from a positivist philosophical position.

Einstein could never accept this interpretation, and firmly stuck to his belief in a physical reality existing independent of our consciousness. And in that sense he held that the theory of quantum mechanics, in spite of all its successes in explaining various physical phenomena and predicting the outcome of experiments, is still an incomplete theory. The completion will come only when it throws light on the nature of physical reality.

In 1935, Einstein teamed up with Boris Podolsky and Nathan Rosen to publish a paper in which he demonstrated how the prevailing ideas of quantum mechanics led to a paradox (now called the EPR paradox): a pair of particles would be able to instantly communicate with each other over long distances.

The same year, Schrödinger published a paper to demonstrate the absurdity of supposing that a system can stay in an "undecided" state until we observe it. He proposed a thought experiment in which a cat is enclosed in a box that contains a radioactive element. When the radioactive element decays, an instrument detects the radiation and opens a vial containing a lethal poison that kills the cat. Now, according to the Copenhagen interpretation of quantum mechanics, the radioactive atom will be in an "undecided" state – a superposition of the 'not decayed' state and the 'decayed' state – until we observe it.



A group photograph of the participants of the 1927 Solvay conference

Schrödinger asks: Will the cat also be in a superposition of the 'alive' state and the 'dead' state until we open the box and observe it?

Most scientists shoved these objections under the rug and proceeded with their business.

Towards the end of his life Einstein became increasingly isolated as the Copenhagen interpretation became 'mainstream', with a generation of scientists being taught to use quantum mechanics to calculate without bothering about its philosophical implication. Yet he stuck steadfastly to his materialistic views. Einstein's biographer Abraham Pais observed "We often discussed his notions on objective reality. I recall that during one walk Einstein suddenly stopped, turned to me and asked whether I really believed that the moon exists only when I look at it."

Since the 1980s there has been a resurgence of interest in the foundations of quantum mechanics. Though all the experiments performed so far confirm the predictions of quantum mechanics, many scientists now feel that the question is still wide open, almost nine decades after it was raised.

The famous physicist and mathematician Roger Penrose who wrote in the foreword of the book *Einstein's miraculous year* : "Why, when Einstein started from a vantage point so much in the lead of his contemporaries with regard to understanding quantum phenomena, was he nevertheless left behind by them in the subsequent development of quantum theory? ··· Many would hold that Einstein was hampered by his 'outdated' realist standpoint, whereas Niels Bohr, in particular, was able to move forward simply by denying the very existence of such a thing as "physical reality" at the quantum level of molecules, atoms, and elementary particles. Yet it is clear that the fundamental advances that Einstein was able to achieve in 1905 depended crucially on his robust adherence to a belief in the actual reality of physical entities at the molecular and sub-molecular levels." Penrose continues to add "Can it really be true that Einstein, in any significant sense, was as profoundly 'wrong' as the followers of Bohr might maintain? I do not believe so. I would, myself, side strongly with Einstein in his belief in a sub-microscopic reality, and with his conviction that present-day quantum mechanics is fundamentally incomplete." Penrose is not alone in this conviction, evidenced by the fact that the foundations of quantum mechanics is still an active area of research that draws inspiration from Einstein's philosophical arguments.

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